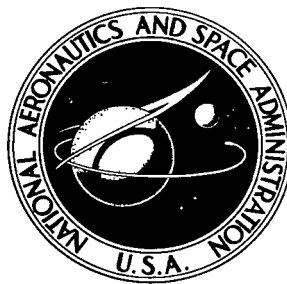


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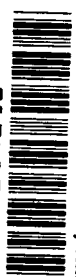
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# THEORETICAL AND SUBJECTIVE BIT ASSIGNMENTS IN TRANSFORM PICTURE PROCESSING

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THEORETICAL AND SUBJECTIVE BIT ASSIGNMENTS IN  
TRANSFORM PICTURE PROCESSING

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SUMMARY

It is shown that all combinations of symmetrical input distributions with difference distortion measures give a bit assignment rule identical to the well-known rule for a Gaussian input distribution with mean-square error. Published work is examined to show that the bit assignment rule is useful for transforms of full pictures, but subjective bit assignments for transform picture coding using small block sizes are significantly different from the theoretical bit assignment rule. An intuitive explanation is based on subjective design experience, and a subjectively obtained bit assignment rule is given.

INTRODUCTION

In transform picture coding, a picture or smaller subpictures are linearly transformed and the resulting transform coefficients are quantized and transmitted. If the picture elements are highly correlated, most of the transform domain energy is concentrated in a few transform vectors that represent averages of adjacent picture elements. The transmitted information is used to specify these high-energy vector coefficients, and the resulting transmission rate for a given distortion is closer to the theoretical bound than the original untransformed picture data rate. Although the Karhunen-Loeve or eigenvalue transformation is best, the computationally simpler Fourier, cosine, or Hadamard transforms have been found useful (refs. 1 to 3).

The bit assignment and quantizers that give minimum mean-square error for a set of independent Gaussian random variables are well known (refs. 4 and 5). The assumptions of Gaussian distribution and mean-square-error criterion are not always correct for the transform coefficients of images, and new quantizers have been found for more appropriate distributions and more general error criteria (refs. 6 and 7). It will be shown that all combinations of symmetrical input distribution with difference distortion measures give a bit assignment algorithm identical to the well-known algorithm for a Gaussian input distribution and mean-square-error criterion. Published work is examined to show that, although the bit assignment algorithm is useful for transforms of full pictures, subjectively designed bit assignments for pictures transformed using small subpicture blocks differ significantly from the theoretical algorithm. Finally, an intuitive explanation of this discrepancy is given, based on a

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\*National Research Council Postdoctoral Research Associate.

subjective design of a Hadamard transform video compressor (ref. 8), and a subjectively obtained bit assignment rule will be stated.

## THEORETICAL BIT ASSIGNMENTS

Huang and Schultheiss (ref. 5) determined the optimum bit assignment using the rate distortion function for Gaussian input and mean-square error. If we assume that the distortion depends on the difference between the exact transform coefficient ( $x_i$ ) and its corresponding representative value ( $y_i$ ), the lower bound of the rate distortion function is well known (refs. 9 and 10). For input distribution  $p(x)$  and distortion

$$D = \frac{1}{N} \sum_{i=1}^N d(x_i - y_i) = \frac{1}{N} \sum_{i=1}^N |x_i - y_i|^q$$

the rate distortion function is bounded as follows:

$$R(D) \geq H[p(x)] - \log_2 D^{1/q} - \log_2 \left[ \frac{2(qe)^{1/q} \Gamma(1/q)}{q} \right]$$

If we assume that the transform coefficients ( $x_i$ ) have the same distribution, differing only in scale or variance, we can show that the entropies of the input distributions  $H[p(x)]$  are identical except for a function of variance. Suppose that  $k(|y|)$  is a probability density having unit variance;  $k(|y|)$  is obviously zero mean and symmetrical about zero. If

$$p(\alpha, x) = \alpha k(|\alpha x|)$$

where  $\alpha$  is a scale parameter, then

$$\int_{-\infty}^{\infty} \alpha k(|\alpha x|) dx = \int_{-\infty}^{\infty} k(|y|) dy = 1$$

Therefore,  $p(\alpha, x)$  is also a zero mean, symmetrical probability density. The variance of  $p(\alpha, x)$  is a function of the scale parameter:

$$\begin{aligned} \sigma^2(\alpha) &= \int_{-\infty}^{\infty} x^2 \alpha k(|\alpha x|) dx \\ &= \int_{-\infty}^{\infty} \frac{y^2}{\alpha^2} \alpha k(|y|) \frac{1}{\alpha} dy \\ &= \frac{1}{\alpha^2} \int_{-\infty}^{\infty} y^2 k(|y|) dy \\ &= \frac{1}{\alpha^2} \end{aligned}$$

The last step follows from the assumption that  $k(|y|)$  has unit variance. The entropy of the distribution is

$$\begin{aligned}
H[p(\alpha, x)] &= - \int_{-\infty}^{\infty} \alpha k(|\alpha x|) \log_2 [\alpha k(|\alpha x|)] dx \\
&= - \int_{-\infty}^{\infty} \alpha k(|y|) \log_2 [\alpha k(|y|)] \frac{1}{\alpha} dy \\
&= - \int_{-\infty}^{\infty} k(|y|) \log_2 \alpha dy - \int_{-\infty}^{\infty} k(|y|) \log_2 [k(|y|)] dy \\
&= -\log_2 \alpha - H[k(|y|)] \\
&= \log_2 \sigma - H[k(|y|)]
\end{aligned}$$

The entropies of a family of input distributions differing in scale are identical except for a function of the scale or variance.

The lower bound on the rate distortion function is then

$$R(D) \geq \log_2 \sigma - H[k(|y|)] - \log_2 D^{1/q} - \log_2 \left[ \frac{2(qe)^{1/q} \Gamma(1/q)}{q} \right]$$

For any fixed symmetrical input distribution and for any fixed difference distortion measure,

$$R(D) \geq \log_2 \frac{\sigma K}{D^{1/q}}$$

where  $K$  is a constant determined by the input distribution and distortion measure. In certain cases, such as the Gaussian distribution with mean-square error or the two-sided exponential distribution with magnitude error, the rate-distortion function is equal to this lower bound (ref. 9, pp. 95 and 99). The parameters and performance of the optimum quantizers have been determined from the Gaussian, two-sided exponential, and gamma input distributions and for the magnitude and mean-square-error criteria (refs. 4, 6, and 7). The resulting rate distortion performance curves are reasonably close to the theoretical rate distortion curves found above (ref. 7). The two-sided exponential and Gaussian densities are similar to experimental distributions of transform coefficient distributions.

For a given theoretical or empirical rate distortion curve, the optimum bit assignment is found after Huang and Schultheiss (ref. 5, p. 293). For  $N$  variables, the rate distortion function is

$$b_i = R(D_i) = \log_2 K \frac{\sigma_i}{D_i^{1/q}}$$

where  $K$  is a constant depending on the input distribution and distortion measure. For an average rate  $A$ ,

$$\sum_{i=1}^N b_i = NA$$

The total distortion is

$$D = \frac{1}{N} \sum_{i=1}^N D_i = \frac{1}{N} \sum_{i=1}^N (K\sigma_i^2 2^{-b_i})^q$$

To minimize distortion under the average rate constraint,

$$\frac{\partial}{\partial b_i} \left( \frac{1}{N} \sum_{i=1}^N K^q \sigma_i^q 2^{-qb_i} + B \sum_{i=1}^N b_i \right) = 0$$

$$K^q \sigma_i^q 2^{-qb_i} = \frac{NB}{q(\ln 2)} = D_i = D$$

Since this quantity is the distortion for the  $i$ th variable, all variables have equal distortion in the optimum bit assignment:

$$b_i = \frac{1}{q} \log_2 \left[ \frac{q(\ln 2) K^q \sigma_i^q}{NB} \right]$$

Using the sum of  $b_i$ ,

$$\sum_{i=1}^N b_i = \sum_{i=1}^N \frac{1}{q} \log_2 \left[ \frac{q(\ln 2) K^q \sigma_i^q}{NB} \right] = NA$$

$$\frac{1}{q} \log_2 \left[ \frac{q(\ln 2) K^q}{NB} \right] = A - \frac{1}{N} \sum_{i=1}^N \log_2 \sigma_i$$

Therefore,

$$b_i = \log_2 \sigma_i + A - \frac{1}{N} \sum_{i=1}^N \log_2 \sigma_i$$

or

$$b_i = 1.66 \log_{10} \sigma_i^2 + C$$

for constant  $C$ . The theoretical bit assignment has the same dependence on variance, and no dependence on the actual distribution or distortion measure, for all combinations of a scaled symmetrical input distribution with a difference distortion measure. Furthermore, since actual quantizer rate distortion performance curves have a slope similar to the slope of the theoretical bound, sometimes differing only by a constant (refs. 4, 7, and 11), the actual quantizers use a very similar bit assignment rule. Note that variables of small variance are not transmitted ( $b_i = 0$ ) when the resulting distortion is less than the equal distortion  $D$  defined above.

Davisson (ref. 3) indicated that objections have been raised to the use of the standard rate distortion function because the source densities are not Gaussian and because mean-square error is not appropriate. The result found here eliminates these objections, at least in bit assignment, for a wide range of source densities and distortion measures. The optimum quantizers are derived directly from the correct-source density and distortion measure (refs. 4, 6, and 7).

### SUBJECTIVE BIT ASSIGNMENTS

Although it is generally accepted that rate distortion theory is not useful in image coding, Mannos and Sakrison (ref. 12) performed an experiment that significantly increases the usefulness of the theory. Their procedure was as follows:

- (1) A nonlinear weighting was performed on the picture samples.
- (2) A smoothed, isotropic power spectral density was estimated.
- (3) The spectral density and rate-distortion theory were used to define the equal maximum distortion in the spectral domain.
- (4) The fast Fourier transform was taken of a full 512 by 512 picture.
- (5) Only the coefficients that exceeded the minimum distortion were transmitted, with Gaussian noise added to give the equal minimum distortion.
- (6) The inverse transform and nonlinear operation were applied.
- (7) The picture was subjectively evaluated.

Although added Gaussian noise, which corresponds to an ideal quantizer, was used rather than an actual quantizer, the above steps conform to the bit assignment theory. The estimated power density and the actual transform coefficients were weighted using different functions and the results evaluated subjectively. Mannos and Sakrison's best curve of contrast sensitivity versus spatial frequency has a peak at 8 cycles/degree and decreases rapidly on either side of the peak, which agrees very well with direct measurements. It follows that, if the coefficient variances are weighted according to spatial frequency or transform vector visibility, the bit assignment theory should produce good subjective results.

For correlated picture elements, the coefficient power decreases as spatial frequency increases. Therefore, weighting according to frequency may be approximated with weighting according to power. Davisson (ref. 3) considered the coefficient distortion to be

$$d_i = \min(\sigma_i^2, B\sigma_i^{2x})$$

The bit assignment is

$$b_i = \frac{1}{2} \log_2 \frac{\sigma_i^2}{\min(\sigma_i^2, B\sigma_i^{2x})} + C$$

where  $C$  adjusts the total number of bits, as usual. The best results for a  $256 \times 256$  Fourier transform picture at 0.5 bits/sample were obtained for  $x = 1/3$ :

$$b_i = \frac{1}{2} \log_2 \sigma_i^{2(1-x)} + C' , \text{ for } C\sigma_i^{2x} > \sigma_i$$

$$b_i = 1.11 \log_{10} \sigma_i^2 + C' , \text{ for } C\sigma_i^{2x} > \sigma_i$$

The bit representation for lower powers or higher spatial frequencies is increased for a fixed total of bits, while the bit representation for high powers is decreased. At viewing distances three times the picture height, the picture subtends  $19^\circ$ , and the picture elements occur 256/19 or 13.5/deg. Thus the spatial components had frequencies from 0.053 for the full picture to  $\sqrt{2}$  (13.5) = 19.0 for a checkerboard of the picture elements. These frequencies lie largely in the region where contrast sensitivity is increasing with spatial frequency, so Davisson's bit reassignment agrees with the work of Mannos and Sakrison.

Landau and Slepian (ref. 1) subjectively quantized  $4 \times 4$  Hadamard transform vectors for  $256 \times 256$  pictures. At a viewing distance three times the picture height, the spatial frequencies of the  $4 \times 4$  Hadamard blocks vary from 256/19  $(1/4)\sqrt{2} = 4.77$  to 256/19  $\sqrt{2} = 19.0$  cycles/deg. Landau and Slepian's bit assignments are shown in figure 1, with several theoretical results based on their covariance values. All bit assignments total 32 bits. The Shannon theory bit assignment and the modified assignment of Kurtenbach and Wintz (ref. 13) are similar, but both are quite different from the empirical assignment. Table I gives estimated spatial frequencies for the Hadamard vectors and shows the application of Mannos and Sakrison's frequency weighting to the vector powers. This bit reassignment is slightly closer to Landau and Slepian's empirical bit assignment. A linear regression on  $b_i$  versus  $\sigma_i^2$  for Landau and Slepian's assignment gives a slope of 3.65, with  $C = 7.10$  for 32 bits. The theoretical and empirical bit assignments differ widely.

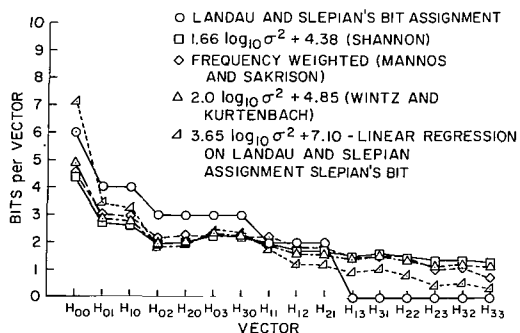


Figure 1.- Comparison of theoretical and empirical bit assignments.

Knauer (ref. 14) implemented real-time video three-dimensional Hadamard transform compression. The video frames are  $512 \times 512$  samples;  $4 \times 4 \times 4$  subpictures with a time dimension are used. Experimentation with a programmable quantizer indicated that Landau and Slepian's compression scheme can be improved very little. In the 1.1-bit/sample bit assignment of Knauer's table II, the vectors representing no temporal change have the same bit assignments as in Landau and Slepian, except that the  $H_{00}$  vector has seven bits.

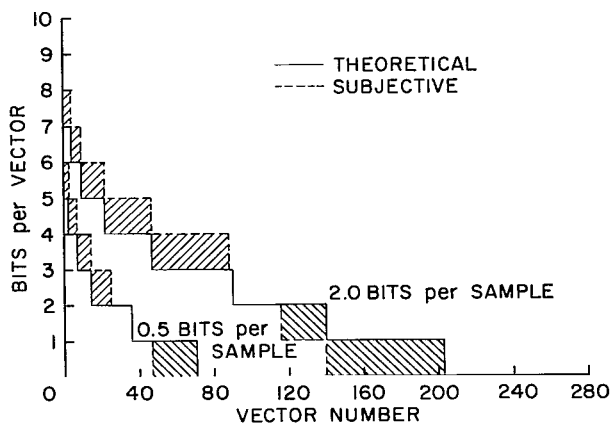


Figure 2.- Theoretical and subjective bit assignments from Habibi and Wintz--(ref. 2).

Habibi and Wintz (ref. 2) encoded four pictures, of  $256 \times 256$  samples, using Karhunen-Loeve, Fourier, and Hadamard transforms in  $16 \times 16$  subpictures. They selected rates of 2.0, 1.0, and 0.5 bits/sample and a bit assignment rule with a slope of 2.0. For the Karhunen-Loeve transform, they found that subjective quality was "improved slightly by assigning more bits to the samples with larger variances and proportionally fewer bits to the samples with the smaller variances," and found that "the modified bit allocation also resulted in a slightly smaller mean-square error." Their theoretical and subjective bit assignments for 2.0 and 0.5 bits/sample are given here in figure 2, and

the reassignment for 1.0 bits is similar. Since the theoretical slope is 2.0, the slope of  $\sigma_i^2$  versus vector number is determined by the theoretical bit assignments (table II). The subjective bit assignments then determine the subjective slope. Taking each group of vectors as a data point, linear regression shows that the subjective slopes are 2.55, 2.59, and 2.68 for the 2, 1, and 0.5 bit/sample bit assignments. This change of slope is similar to, but less than, that of Landau and Slepian. Habibi and Wintz report that no reassignments that were consistently better for the Fourier and Hadamard transforms were found. The frequency range for  $16 \times 16$  Hadamard blocks of  $256 \times 256$  pictures, viewed at a distance three times the height, varies from 1.19 to 19.0 cycles/deg. Assuming that the Karhunen-Loeve vectors have a similar frequency range, the bit assignment for higher power vectors would be reduced from the theoretical, as in Davisson's experiment discussed previously. The subjective bit reassignment is contrary to the indication of subjective frequency weighting.

It is apparent that subjective bit assignments differ significantly from theoretical bit assignments. The work of Mannos and Sakrison (ref. 12) and Davisson (ref. 3) indicates that, for transforms of complete  $256 \times 256$  pictures, subjective frequency weighting accounts for the discrepancy. The work of Landau and Slepian (ref. 1), Knauer (ref. 14), and Habibi and Wintz (ref. 12), for small subpictures, shows that larger vectors should be assigned relatively more bits. For small blocks, the reassignment is in excess of, or in contradiction to, the effect of subjective frequency weighting.

#### INTUITIVE EXAMINATION OF BIT ASSIGNMENT PROBLEM

The reason that subjective bit assignments differ from theoretical bit assignments becomes apparent when we consider a subjective compression design procedure for small block size. The  $4 \times 4$  Hadamard vectors are shown in frequency order in figure 3. If we start with a transformed picture of highest

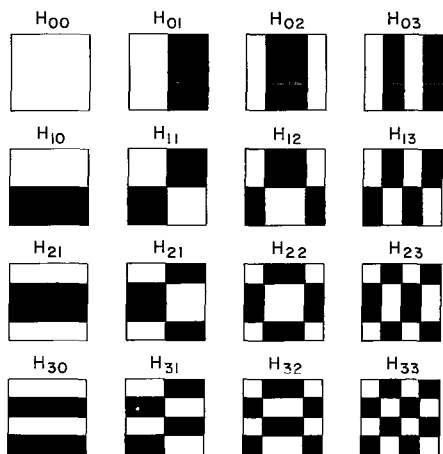


Figure 3.- Sequence-ordered  $4 \times 4$  Hadamard basis vectors.

quality, all transform coefficients have closely spaced representative values covering the full possible range. In the first attempt to reduce the rate, some vectors are represented by a zero value to determine which can be eliminated with least effect. Usually, many high-frequency vectors have such small variance that this causes little degradation. Next, the ranges of the remaining vectors are reduced until edge blurs appear. Lastly, representative values are thinned out of the retained vector ranges until granular noise due to the coarse quantization becomes objectionable. Each final vector representation has sufficient range to adequately reproduce edges and has enough intermediate levels to minimize quantizing

noise. These tests of quality are extremely subjective, but they have some correlation with mean-square error.

In a picture of high contrast, some of the subpictures will be very light or very dark, so that the block average transform vector usually has the full possible range (refs. 1 and 14). Suppose there are high contrast edges in the picture. Some portions of these edges will coincide with block edges, and will be reproduced with the full original contrast. Other portions of the same edge will intersect subpictures, causing transform vector coefficients of extreme value. These values must be correctly represented or a single edge will have varying contrast and sharpness. (Edge ghosts also affect the vector elimination decision, as described by Knauer (ref. 15).) Adequately representing the edges requires that some of the vector coefficients have wider representative value ranges and more bits than typical subpictures would need.

The above argument can be quantified if some assumptions are made. For an  $N \times N$  subpicture, the Hadamard transform matrix,  $H$ , is an  $N^2 \times N^2$  matrix of  $\pm 1$  elements. The vector coefficient value is found by adding and subtracting sample values. The  $H$  matrix is multiplied by  $1/N$ , so that total power is equal in the sample and transform domains. If the sample correlation matrix is  $\overline{x^T x}$ , the transform correlation matrix is

$$\overline{y^T y} = \frac{1}{N} H \overline{x^T x} \frac{1}{N} H$$

The powers or variances of the transform vectors are on the diagonal of  $\overline{y^T y}$ . We assume unit variance samples produced by a first-order Markov process with sample to sample correlation of 0.95, vertically and horizontally (refs. 2 and 3). The equation above defines the transform coefficient variances, which determine the theoretical bit assignment and give an estimate of the coefficient range. Quantizers for the Gaussian distribution have a maximum representative value of about  $3\sigma$  (ref. 4). Average values of the  $3\sigma$  point for Hadamard

vectors and groups of Hadamard vectors of different block size are given in table III.

We next determine the largest vector coefficients produced by sharp, high-contrast edges. If the unit variance samples are assumed to have uniform distributions, the sample range is  $\pm\sqrt{3}$ . Consider a vertical edge dividing a light ( $+\sqrt{3}$ ) region on the left from a dark ( $-\sqrt{3}$ ) region on the right. The value of  $H_{00}$  for a  $4 \times 4$  subpicture (fig. 3) varies from  $+4\sqrt{3}$  for all light samples to  $-4\sqrt{3}$  for all dark samples, since  $16(N^2)$  samples are added with the same sign and divided by a scale factor of  $(1/4), (1/N)$ . When the transition of the  $H_{01}$  vector lines up with the edge, the  $H_{01}$  coefficient is  $4\sqrt{3}$ ; the value should be  $-4\sqrt{3}$  for an edge transition of opposite sign. When the edge coincides with one of the transitions of  $H_{02}$ , the coefficient is  $-2\sqrt{3}$ , and  $H_{03}$  has the same extreme edge induced value. The same maximum values occur for horizontal edges and the corresponding vectors with horizontal transitions, and similar extreme values can be found for diagonal and skew edges and Hadamard vectors with both horizontal and vertical transitions. The maximum-edge-induced coefficient values for various block sizes are shown in table III. If the subpicture exactly matched the Hadamard vector patterns, and had maximum light-dark range, the coefficient value would be  $N\sqrt{3}$ , the maximum value for all coefficients.

For the smaller block sizes shown in the table, the maximum-edge-induced coefficients of vectors with all vertical or all horizontal transitions ( $H_{0x}$  or  $H_{x0}$ ) are equal to 9 to  $13\sigma$ , while the maximum-edge-induced coefficients of vectors with both horizontal and vertical transitions are equal to 3 to  $5\sigma$ . The existence of edges requires a larger extension of range for vectors with only vertical or horizontal transitions than for vectors with both. For small block sizes, all samples are highly correlated and the  $H_{00}$  vector has the sample distribution, assumed to be uniform and full range. Then the variance is approximately  $(\sqrt{3}N)^2/3 = N^2$  and  $3\sigma = 3N$ .

The  $H_{00}$  vector should have full range, and if maximum-edge-induced coefficients occur, the  $H_{01}$  and  $H_{10}$  should have full range, the  $H_{02}$ ,  $H_{03}$ ,  $H_{20}$ , and  $H_{30}$  should have half range, etc. As shown in table IV, the largest representative values in three subjective  $4 \times 4$  Hadamard systems are 20 to 50 percent of the maximum edge coefficients. The edge differences are usually only from light to dark rather than from white to black. Experimental Hadamard coefficient distributions are usually exponential, with the number of extreme values depending on the frequency and sharpness of edges. The observed high-value coefficients due to edges, though less than the maximum possible, still require representative values extending to the theoretical 5 or 6 $\sigma$  point.

In the design of a real-time hardware Hadamard transform video compressor, an adaptive system was used (ref. 8). Depending on the vector coefficient magnitudes,  $4 \times 4$  blocks were quantized differently. The most frequently used (61 percent) bit assignment and quantization was based on a theoretical bit assignment for a Markov correlation model with 0.95 correlation between horizontally adjacent elements. The second method was used when vector coefficients exceeded the ranges in the first method, and is quite similar to Landau and Slepian's design. The third method is used when vector magnitudes exceed

the range of the second method, and provides superior reproduction of sharp, high-contrast edges. It is apparent that edges, rather than the most typical subpictures, determined Landau and Slepian's design.

As block size increases from  $2 \times 2$  to  $8 \times 8$ , the maximum-edge-induced coefficients become smaller compared to the theoretical standard deviation. For full picture transforms, large-scale edges similarly can cause extreme coefficient values, but several factors make this possibility relatively unimportant. For a full  $512 \times 512$  picture, there is only one  $H_{01}$  vector. If the picture is broken into  $4 \times 4$  subpictures, there are 16,384. The observed frequency of large  $H_{01}$  coefficients for full-picture transforms will be one in many pictures; for  $4 \times 4$  transforms the observed frequency will be many in one picture. Most compression systems are tested on only a few pictures, and possibly full-picture systems would poorly reproduce a picture half black and half white (i.e., a picture equal to an extreme  $H_{01}$  vector). A second reason that edge effects are unimportant in full-picture transforms is that, since each coefficient describes the full picture, there are no noticeable differences in edge rendition between regions. A blurred edge is uniformly blurred, and such blurs are less noticeable because comparison is difficult. Thirdly, all full-picture vector coefficients are determined using all picture elements ( $2^{18}$ ), which tends to average local features or edges and make the coefficient distribution approach the Gaussian, which has relatively fewer extreme values than the exponential distribution observed for small subpictures.

The subjective bit assignment rule for small block size seems to be:

- (1) Quantize the block average coefficient uniformly over the full possible range, with 6 to 8 bits.
- (2) Determine the theoretical bit assignment for the required bit rate, using measured variances or the theoretical variances for typical pictures.
- (3) Add one bit (and extend the representative value range) for vectors with all vertical or all horizontal transitions.
- (4) Use zero bits instead of the theoretical number for the highest sequency vectors until the number bits is reduced to the required rate.

This rule reproduces Landau and Slepian's bit assignment in figure 1, while the closest bit assignment based on coefficient variances is a poor approximation and has no intuitive justification.

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National Aeronautics and Space Administration

Moffett Field, Calif., 94035, October 19, 1976

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TABLE I.- FREQUENCY WEIGHTING OF LANDAU AND SLEPIAN'S VARIANCES

Vector	Designation	Variance, <sup>a</sup> $\sigma^2$	Frequency	A(fr) <sup>b</sup>	$1.66 \log_{10} \sigma^2 A^2 + 4.81$	Bits <sup>c</sup>
1	H <sub>00</sub>	1.00	4.77	0.88	4.50	6
2	H <sub>01</sub>	.098	7.55	.98	2.98	4
3	H <sub>10</sub>	.087	7.55	.98	2.90	4
4	H <sub>02</sub>	.035	10.67	.93	2.15	3
5	H <sub>20</sub>	.038	10.67	.93	2.22	3
6	H <sub>03</sub>	.051	13.92	.79	2.20	3
7	H <sub>30</sub>	.048	13.92	.79	2.15	3
8	H <sub>11</sub>	.034	9.55	.96	2.18	2
9	H <sub>12</sub>	.024	12.17	.87	1.78	2
10	H <sub>21</sub>	.024	12.17	.87	1.78	2
11	H <sub>13</sub>	.020	15.09	.74	1.43	0
12	H <sub>31</sub>	.022	15.09	.74	1.49	0
13	H <sub>22</sub>	.019	14.31	.77	1.43	0
14	H <sub>23</sub>	.015	16.88	.65	1.02	0
15	H <sub>32</sub>	.016	16.88	.65	1.07	0
16	H <sub>33</sub>	.014	19.09	.54	.71	0

<sup>a</sup>Landau and Slepian, table I (ref. 1).

<sup>b</sup>Mannos and Sakrison, equation (23) (ref. 12).

<sup>c</sup>Landau and Slepian, table III (ref. 1).

TABLE II.- HABIBI-WINTZ THEORETICAL AND EMPIRICAL BIT ASSIGNMENTS<sup>a</sup>

Vectors	Theoretical bits, $b_{ith}$	Subjective bits, $b_i$	Approximation, $b_{ia}$
2 bits/sample; $b_{ia} = 2.55(b_{ith}/2.0) - 0.61$			
1-3	7	8	8.32
4-9	6	7	7.04
10-21	5	6	5.77
22-47	4	5	4.49
47-88	3	4	3.22
89-90	3	3	3.22
91-115	2	2	1.94
116-139	2	1	1.94
140-203	1	0	.67
204-256	0	0	-.61
1 bit/sample; $b_{ia} = 2.59(b_{ith}/2.0) - 0.48$			
1-2	6	7	7.27
3-6	5	6	5.96
7-15	4	5	4.69
16-38	3	4	3.40
39-56	2	3	2.10
57-66	2	2	2.10
67-73	2	1	2.10
74-122	1	0	.81
123-256	0	0	-.48
0.5 bit/sample; $b_{ia} = 2.68(b_{ith}/2.0) - 0.73$			
1-2	5	6	5.98
3-6	4	5	4.64
7-14	3	4	3.29
15-25	2	3	1.95
26-35	2	2	1.95
36-46	1	1	.61
47-71	1	0	.61
72-256	0	0	-.73

<sup>a</sup>Habibi and Wintz (ref. 2)

TABLE III.-  $3\sigma$  AND MAXIMUM-EDGE-INDUCED VALUES FOR HADAMARD TRANSFORM COEFFICIENTS

Vector	Block size						
	$2 \times 2$		$4 \times 4$		$8 \times 8$		$N \times N$
	$3\sigma$	Edge Maximum	$3\sigma$	Edge Maximum	$3\sigma$	Edge Maximum	Edge Maximum
$H_{00}$	5.87	3.46	11.41	6.93	21.64	13.85	$N\sqrt{3}$
$H_{01}, H_{10}$	.79	3.46	1.86	6.93	4.82	13.85	$N\sqrt{3}$
$H_{02}, H_{03}, H_{20}, H_{30}$	-	-	.92	3.46	2.23	6.93	$(N/2)\sqrt{3}$
$H_{04}-H_{07}, H_{40}-H_{70}$	-	-	-	-	1.04	3.46	$(N/4)\sqrt{3}$
$H_{11}$	.52	.87	.95	1.73	2.37	3.46	$(N/4)\sqrt{3}$
$H_{12}, H_{13}, H_{21}, H_{31}$	-	-	.64	.87	1.38	1.73	$(N/8)\sqrt{3}$

TABLE IV.- FRACTION OF RANGE REPRESENTED IN SUBJECTIVE  $4 \times 4$  HADAMARD DESIGNS AND MAXIMUM-EDGE-INDUCED FRACTION OF RANGE

Vectors	Fraction of full range represented			Edge maximum value divided by full range ( $N\sqrt{3}$ ).
	Landau & Slepian (ref. 1)	Knauer (ref. 14) (Vectors with no time component)	Jones (ref. 8) (Mode 3 for extreme edges)	
$H_{00}$	0.94	0.99	0.98	1.0
$H_{01}, H_{10}$	.24	.31, 0.19	.48, 0.23	1.0
$H_{02}, H_{03}, H_{20}, H_{30}$	.093	.093, 0.074	.23, 0.11	0.5
$H_{11}$	.069	.090	.11	0.25
$H_{12}, H_{21}$	.069	.069	0	0.125
All others	0	0	0	0.125-0.063



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